Fast Fourier Transform

1 FAST FOURIER TRANSFORMS

Write the Fourier transforms of the following vectors in terms of two Fourier transforms with half the length (as in the fast Fourier transform method).

A) $\mathbf{f} = [-3, -2, -1, 0, 1, 2, 3, 4]$

B) $\mathbf{f}_e = [-3, -1, 1, 3]$

C) $\mathbf{f}_o = [-2, 0, 2, 4]$

D) $\mathbf{f}_{ee} = [-3, 1]$

Compute the Fourier transforms of the length 2 vectors resulting from (B) and (C), and combine them to get the Fourier transform of the vector in (A).

E) $\mathbf{g} = [0, 1, 0, -1, 0, 1, 0, -1]$

F) $\mathbf{g}_e = [0, 0, 0, 0]$

G) $\mathbf{g}_o = [1, -1, 1, -1]$

H) $\mathbf{g}_{oe} = [1, 1]$

I) $\mathbf{g}_{oo} = [-1, -1]$

Compute the Fourier transforms of the length 2 vectors resulting from (F) and (G), and combine them to get the Fourier transform of the vector in (E).

2 THEORY AND ADVANCED PROBLEMS

A) The fast Fourier transform can also be written in terms of matrices. It is equivalent to decomposing the Fourier matrix as a product of three other matrices. The first splits a vector into even and odd parts, the second does Fourier transform on each of these parts, and the third recombines the results to get the Fourier transform of the original vector.

What is the decomposition?

B) When computing using the fast Fourier transform, you actually only need to ever calculate the first half of the coefficients, since the second half are conjugate.

Similarly, when computing the inverse fast Fourier transform, you can solve using only the first half of the coefficients. Convert the inverse fast Fourier formula to only use the first half of the Fourier coefficients.